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RESEARCH PROBLEMS

Problem 97. Posed by J.-C. Bermond.
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Let G be a regular graph with edge-set $E(G)$. The graph G is said to have a Hamilton decomposition if either its degree is even and $E(G)$ can be partitioned into Hamilton cycles or its degree is odd and $E(G)$ can be partitioned into Hamilton cycles and a single 1-factor.

Conjecture. If G has a Hamilton decomposition, then its line graph can be decomposed into Hamilton cycles.

There are a few results dealing with the conjecture. A. Kotzig has proved that if G is a cubic graph with a Hamilton decomposition (i.e. it has a Hamilton cycle), then its line graph can be decomposed into two Hamilton cycles. Jaeger [1] proved that if $|E(G)|$ is even and G can be decomposed into Hamilton cycles, then the line graph of G has a 1-factorization. The key to the proof lies in the following result: If G can be decomposed into two Hamilton cycles, then its line graph can be decomposed into three Hamilton cycles.

The case when G is the complete graph seems to be unsolved. It is a particular case of a conjecture of A. Kotzig saying that the complete graph has a compatible eulerian decomposition.

Reference

[1] F. Jaeger, The 1-factorization of some line graphs, Discrete Math. 46 (1983) 89–92.

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A digraph is said to be *strong* if for any ordered pair (u, v) of vertices, there is a directed path from u to v. The *independence number* of a digraph G, denoted $\alpha(G)$, is the maximum cardinality of a set of vertices no two of which are joined by an arc of G.

Conjecture. The minimum number of directed cycles needed to cover the vertices of a strong digraph G is less than or equal to the independence number of G.

The conjecture is true when $\alpha(G)=1$ because G contains a strong tournament. An old theorem of Camion states that a strong tournament has a directed Hamilton cycle. The conjecture is also true for $\alpha(G)=2$ by a result of Chen and Manalastas [1].

Let $\alpha'(G)$ denote the maximum cardinality of an induced subdigraph without any directed cycles. The conjecture is true when $\alpha(G)$ is replaced with $\alpha'(G)$ [2]. See [3] for a recent survey dealing with the conjecture and generalizations.

References

[1] C.C. Chen and P. Manalastas, Every finite strongly connected digraph of stability 2 has a hamiltonian path, Discrete Math. 44 (1983) 243–250.

[2] M.C. Heydemann, Minimum number of circuits covering the vertices of a strong digraph, Annals Discrete Math. 27 (1985) 287–296.

[3] B. Jackson and O. Ordaz, Chvátal–Erdös conditions for paths and cycles in graphs and digraphs, a survey. Preprint.